Technical Note

An empirical correlation for natural convection heat transfer from thin isothermal circular disks at arbitrary angles of inclination

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1. Introduction

The circular disk geometry is important in a broad spectrum of applications, such as electronic components and silicon disk wafers. The circular disk geometry, however, has received relatively little attention in the archival literature compared to other geometries. Considerable empirical data exist in the literature for natural convection heat transfer involving a variety of geometries, and for various ranges of the Rayleigh number. Current heat transfer textbooks [1,2] present empirical correlations for natural convection heat and mass transfer for horizontal and inclined flat plates and cylinders, spheres, bispheres, oblate and prolate spheroids, horizontal upward and downward facing surfaces, cubes of various orientations, vertical and inclined channels, rotating geometries, as well as geometries within enclosures. A geometry that seems to be missing from these lists is that of a thin circular disk.

Some experimental research has been devoted to natural convection heat transfer from stationary and rotating horizontal circular disk *surfaces*; heated upward facing [3–8] and cooled downward facing [9], both classical and numerical [10–14]. Hassani and Hollands [15] performed experiments measuring the natural convection heat transfer from a *complete* circular disk in both vertical and horizontal orientations. They proposed a characteristic length such that the experimental data obtained could be collapsed with certain other geometrical shapes for a limited range of the Rayleigh number; the goal being a type of universal correlation. Their experimental data were obtained using a single

disk heat transfer model with a diameter of 82 mm. More recently, Kobus and Wedekind [16,17] presented extensive experimental data, and corresponding empirical correlations, for both vertical [16] as well as horizontal [17] thin circular disks. To the best knowledge of the authors, there do not appear to be any models in the archival literature, empirical or otherwise, for predicting the natural convection heat transfer from a complete circular disk geometry at inclination angles between the vertical and horizontal limits. [1] Such is the objective of the current research.

Experiments with air were performed for disks of different diameters and thickness-to-diameter aspect ratios, and at a variety of inclinations between the vertical and horizontal limits. The data obtained in this research are compared with those obtained previously for both the vertical [16] and horizontal [17] orientations, and also with the other limited amount of horizontal and vertical disk data found in the archival literature [15]. The goal of the present research is to develop an empirical correlation in the familiar classical form

$$Nu_d = C Ra_d^n \tag{1}$$

for a thin, stationary, circular disk, including the influence of inclination angle.

As mentioned earlier, the present authors developed a novel approach to obtaining accurate experimental heat transfer data by utilizing commercially available thermistors of circular disk geometry [16,17,28,29]. This

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[1] Some research does appear in the archival literature regarding inclined *surfaces*, under conditions of natural as well as combined forced and natural convective heat transfer, but the research does not appear to include a circular surface or a complete circular disk geometry [18–27].

prior research dealt primarily with vertical and horizontal circular disks and flat plates. Because of the simplicity of the experimental technique, however, the experimental apparatus was readily adapted to handle inclined orientations.

2. Experimental apparatus and measurement techniques

The experimental apparatus and measurement techniques used in the present research were similar to those used in previous research [16]. Briefly, the circular disks that were used as heat transfer models for the present experimental data were commercially available disk-type thermistors, chosen because they provided a unique combination for indirectly measuring the surface temperature and the convective heat transfer rate. The thermistor was self-heated by means of Joule heating. Conduction losses through the thermistor lead wires (0.127 mm diameter) were minimized (less than 8%) by using constantan wire, which has a low enough thermal conductivity to minimize the 'fin effect', and an electrical resistivity low enough to minimize Joule heating in the lead wires themselves. A one-dimensional analysis was carried out on the lead wires modeling the 'fin effect' and taking into account the Joule heating. Even though they

were small, corrections were made for lead losses in obtaining the experimental data [29].

Considerable experimental data were obtained in the present research. Five different circular disk models were tested, ranging in diameter, $5.2 \leqslant d \leqslant 19.97$ mm, and in thickness-to-diameter aspect ratio, $0.063 \leqslant (t/d) \leqslant 0.163$ [16]. The experimental results for the largest of the heat transfer models are depicted in dimensionless form in Fig. 1(a), where the Nusselt number, Nu_d, is plotted as a function of the Rayleigh number, Ra_d. The data indicate that heat transfer coefficients are generally higher for disks in a vertical orientation than for disks in a horizontal orientation, at least over this range of the Rayleigh number. This is not always the case. Fig. 1(b) is the same type of data as Fig. 1(a) except that it is for the smallest of the heat transfer models. The influence of inclination angle that is clear in Fig. 1(a) is diminished for the smaller heat transfer models. Fig. 1(b), which illustrates the experimental data for the lowest range of the Rayleigh number, shows very little difference between the vertical, horizontal, or inclined disk orientations (within experimental uncertainty).

Fig. 2(a) illustrates the cumulative experimental data of all of the heat transfer models. It is clear from this figure that the influence of inclination angle appears to be continuous, and is greatest at the highest Rayleigh
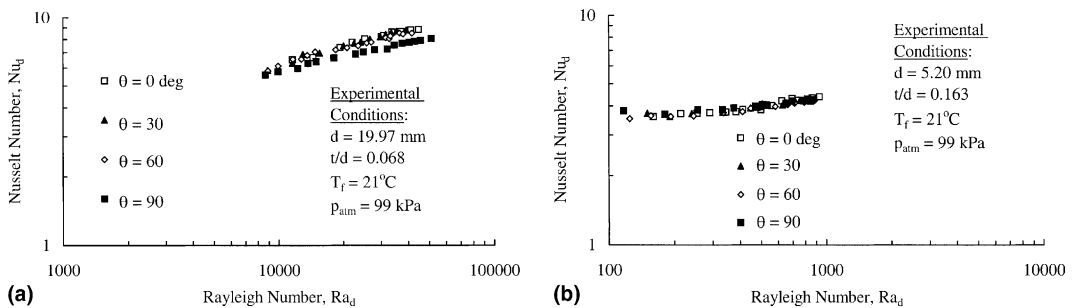

Fig. 1. Natural convection heat transfer for stationary isothermal circular disk, influence of inclination angle: (a) $d = 19.97$ mm; (b) $d = 5.20$ mm.
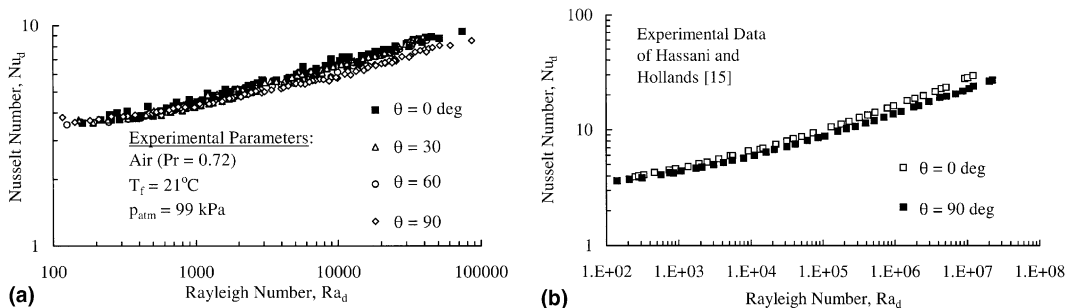

Fig. 2. Natural convection heat transfer for stationary isothermal circular disks, influence of inclination angle: (a) totality of experimental data; (b) data of Hassani and Hollands.

numbers, and least at the lowest. It should be noted that the experimental data of Hassani and Hollands [15] are also included in Fig. 2(a). As was mentioned earlier, their investigation was limited to a single disk model of diameter, $d = 82$ mm, and only included vertical and horizontal orientations. Excellent agreement exists between the experimental data of the present authors and that of Hassani and Hollands [15] for $10^2 \leqslant Ra_d \leqslant 10^5$. The increasing influence of orientation at the higher Rayleigh numbers is clearly evident in the data of Hassani and Hollands shown in Fig. 2(b), even though only vertical and horizontal disk data are presented.

3. Empirical correlation

As mentioned earlier, the present authors previously developed empirical correlations for vertical [16] and horizontal [17] isothermal disks. Interestingly, the coefficient, C, in Eq. (1), was the same for both the horizontal and vertical correlations, only the exponent of the correlation, n, was different. In light of this, empirical correlations were developed in the current research for other angles of inclination. The purpose was to ascertain the influence of inclination angle on the exponent, n. Figs. 3(a) and (b) display both the experimental data and empirical correlation for inclination angles of 30 and 60° from vertical, respectively. From the experimental data in this and previous research [16,17], it was determined that the exponent, n, in the classical Nusselt–Rayleigh correlation, expressed by Eq. (1), varied as a nearly linear function of inclination angle. The correlations in Fig. 3 are valid for $2 \times 10^2 \leqslant Ra_d \leqslant 10^4$. Based on the totality of the data, including that of Hassani and Hollands [15], a generalized correlation was obtained. In addition, because the vertical and horizontal data of Hassani and Hollands [15] extend to a higher Rayleigh number than that of the current research, another correlation has been developed that encompasses the higher Rayleigh numbers. Therefore, the empirical correlation for all of the available experimental data can be expressed as

$$Nu_d = C Ra_d^{a - b\theta}, \tag{2}$$

where the coefficients a, b and C are given in Table 1. The average correlation coefficient for the above expression is 0.987, with a maximum deviation of less than 10%. The result of an analysis of the experimental uncertainty is given by Kobus and Wedekind [16], with a maximum uncertainty of less than 10%, albeit experimental uncertainties would normally be expected to be less than the maximum. The above correlation has the advantage of being valid over the full range of inclination angles between the vertical and horizontal limits. Other research dealing with inclined surfaces [18–27] proposed correlations, or developed numerical codes, that would predict or display experimental data in the form $Nu_x \cos \theta = C f(Re_x)$. This functional form has the disadvantage of not being valid at $\theta = 90°$. The form of Eq. (2) does not have such a limitation.

An alternative correlation is also suggested that would give good results, but at the expense of mathematical complexity. This alternative correlation was developed by Hassani and Hollands [15] and is expressed as

$$Nu_{\sqrt{A}} = \left\{ \left[\left(0.515 Ra_H^{1/4} \right)^8 + \left(0.1 Ra_H^{1/3} \right)^8 \right]^{1.07/8} \right.$$
$$\left. + 3.45^{1.07} \right\}^{1/1.07}, \quad 10^2 \leqslant Ra_H \leqslant 10^8, \tag{3}$$

where A is the surface area, and the characteristics length, H, is a function that involves of the height of the inclined disk as well as the average periphery. For vertical or horizontal disks, the characteristic length, H, can be calculated without significant difficulty. For inclined disks, however, the calculation of H becomes more mathematically complex because of the way the average periphery must be calculated [15]. Therefore, although for inclined circular disks the Nusselt–Rayleigh correlations expressed by Eqs. (2) and (3) yield very similar results, the relative simplicity of Eq. (2) is self-evident.
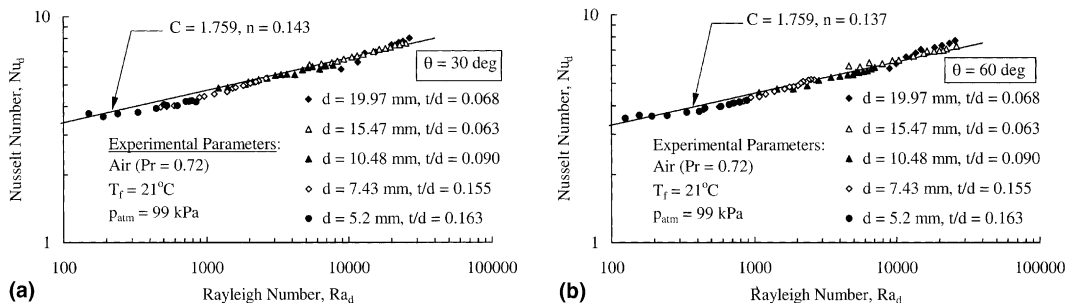

Fig. 3. Natural convection heat transfer for stationary isothermal circular disks, empirical correlations: (a) 30° from vertical; (b) 60° from vertical.

Table 1
Empirically determined coefficients and exponents for Eq. (2); $0 \leqslant \theta \leqslant 90°$

Rayleigh number, Ra_d	C	a	b (deg^{-1})
2×10^2–10^4	1.759	0.150	2.22×10^{-4}
10^4–3×10^7	0.9724	0.206	1.33×10^{-4}

4. Summary and conclusions

The significant volume of experimental data presented agreed well with the proposed empirical correlation over a wide range of Rayleigh numbers. In addition, the current data agreed with the limited data available in the archival literature for vertical and horizontal orientations. Since only air was tested, the correlations are recommended for Prandtl numbers near unity, which includes most common gases. The correlations may be valid for Prandtl numbers outside this range, however, experimental verification of this is unavailable at this time, but should be the subject of future research. Also, the maximum aspect ratio recommended should not be much more than the maximum of the heat transfer models tested, thus $(t/d) < 0.2$. The influence of larger aspect ratios should also be the subject of future research.

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